The happiness function is

*h*(*r*) = E[*r*] – Var[*r*]

where *r* = *w*1*r*1 + *w*2*r*2

 0 < *w*1, *w*2 < 1

 *w*1 + *w*2 = 1

Using the laws of expectation,

E[*r*] = *w*1E[*r*1] + *w*2E[*r*2] **(I)**

Using the laws of variance,

Var[*r*] = $w\_{1}^{2}$Var[*r*1] + $w\_{2}^{2}$Var[*r*2] + 2*w*1*w*2Cov[*r*1, *r*2] **(II)**

Thus, *h*(*r*) = E[*r*] – Var[*r*] = = *w*1E[*r*1] + *w*2E[*r*2] – $w\_{1}^{2}$Var[*r*1] – $w\_{2}^{2}$Var[*r*2] – 2*w*1*w*2Cov[*r*1, *r*2] **(III)**

Starting from *h*(*r*) = *w*1E[*r*1] + *w*2E[*r*2] – $w\_{1}^{2}$Var[*r*1] – $w\_{2}^{2}$Var[*r*2] – 2*w*1*w*2Cov[*r*1, *r*2], and using the substitution *w*2 = (1 – *w*1), we obtain

*h*(*r*) = *w*1E[*r*1] + (1 – *w*1)E[*r*2] – $w\_{1}^{2}$Var[*r*1] – (1 – *w*1)2 Var[*r*2] – 2*w*1(1 – *w*1)Cov[*r*1, *r*2] (**IV**)

*h*(*r*) = *w*1E[*r*1] + (1 – *w*1)E[*r*2] – $w\_{1}^{2}$Var[*r*1] – (1 – *w*1)2 Var[*r*2] – 2*w*1(1 – *w*1)Cov[*r*1, *r*2]

= *w*1E[*r*1] + E[*r*2] – *w*1E[*r*2] – $w\_{1}^{2}$Var[*r*1] – (1 – 2*w*1 + $w\_{1}^{2}$)Var[*r*2] – (2*w*1 – 2$w\_{1}^{2}$)Cov[*r*1, *r*2]

= *w*1E[*r*1] + E[*r*2] – *w*1E[*r*2] – $w\_{1}^{2}$Var[*r*1] – Var[*r*2] + 2*w*1Var[*r*2] – $w\_{1}^{2}$Var[*r*2]

– 2*w*1Cov[*r*1, *r*2] + 2$w\_{1}^{2}$Cov[*r*1, *r*2]

Regrouping the terms in $w\_{1}^{2}$, *w*1 and constants, we obtain a quadratic in *w*1:

*h*(*r*) = {– $w\_{1}^{2}$Var[*r*1] – $w\_{1}^{2}$Var[*r*2] + 2$w\_{1}^{2}$Cov[*r*1, *r*2]} +

 {*w*1E[*r*1] – *w*1E[*r*2] + 2*w*1Var[*r*2] – 2*w*1Cov[*r*1, *r*2]} +

 {E[*r*2] –Var[*r*2]}

Differentiating with respect to *w*1, we obtain

$\frac{d}{dw\_{1}}[h(r)] $= {–2*w*1Var[*r*1] –2*w*1Var[*r*2] + 4*w*1Cov[*r*1, *r*2]} +

{E[*r*1] –E[*r*2] + 2Var[*r*2] – 2Cov[*r*1, *r*2]}

= *w*1{–2Var[*r*1] –2Var[*r*2] + 4Cov[*r*1, *r*2]} + {E[*r*1] –E[*r*2] + 2Var[*r*2] – 2Cov[*r*1, *r*2]}

Equating $\frac{d}{dw\_{1}}[h(r)]$ to 0 and solving for *w*1, we have

*w*1{–2Var[*r*1] –2Var[*r*2] + 4Cov[*r*1, *r*2]} + {E[*r*1] – E[*r*2] + 2Var[*r*2] – 2Cov[*r*1, *r*2]} = 0

⇒ *w*1{–2Var[*r*1] –2Var[*r*2] + 4Cov[*r*1, *r*2]} = –{E[*r*1] – E[*r*2] + 2Var[*r*2] – 2Cov[*r*1, *r*2]}

⇒ *w*1{–2Var[*r*1] –2Var[*r*2] + 4Cov[*r*1, *r*2]} = – E[*r*1] + E[*r*2] – 2Var[*r*2] + 2Cov[*r*1, *r*2]

and $w\_{1}= \frac{– E[r\_{1}] + E[r\_{2}] – 2Var[r\_{2}] + 2Cov[r\_{1}, r\_{2}]}{–2Var\left[r\_{1}\right] – 2Var[r\_{2}] + 4Cov[r1, r\_{2}]}$